1. A teacher wishes to test whether playing background music enables students to complete a task more quickly. The same task was completed by 15 students, divided at random into two groups. The first group had background music playing during the task and the second group had no background music playing.
The times taken, in minutes, to complete the task are summarised below.

|  | Sample size <br> $n$ | Standard deviation <br> $s$ | Mean <br> $\bar{x}$ |
| :---: | :---: | :---: | :---: |
| With background music | 8 | 4.1 | 15.9 |
| Without background music | 7 | 5.2 | 17.9 |

You may assume that the times taken to complete the task by the students are two independent random samples from normal distributions.
(a) Stating your hypotheses clearly, test, at the $10 \%$ level of significance, whether or not the variances of the times taken to complete the task with and without background music are equal.
(b) Find a 99\% confidence interval for the difference in the mean times taken to complete the task with and without background music.

Experiments like this are often performed using the same people in each group.
(c) Explain why this would not be appropriate in this case.
2. A car manufacturer claims that, on a motorway, the mean number of miles per gallon for the Panther car is more than 70 . To test this claim a car magazine measures the number of miles per gallon, $x$, of each of a random sample of 20 Panther cars and obtained the following statistics.

$$
\bar{x}=71.2 \quad s=3.4
$$

The number of miles per gallon may be assumed to be normally distributed.
(a) Stating your hypotheses clearly and using a 5\% level of significance, test the manufacturer's claim.

The standard deviation of the number of miles per gallon for the Tiger car is 4 .
(b) Stating your hypotheses clearly, test, at the $5 \%$ level of significance, whether or not there is evidence that the variance of the number of miles per gallon for the Panther car is different from that of the Tiger car.
3. A farmer set up a trial to assess whether adding water to dry feed increases the milk yield of his cows. He randomly selected 22 cows. Thirteen of the cows were given dry feed and the other 9 cows were given the feed with water added. The milk yields, in litres per day, were recorded with the following results.

|  | Sample size | Mean | $s^{2}$ |
| :---: | :---: | :---: | :---: |
| Dry feed | 13 | 25.54 | 2.45 |
| Feed with water added | 9 | 27.94 | 1.02 |

You may assume that the milk yield from cows given the dry feed and the milk yield from cows given the feed with water added are from independent normal distributions.
(a) Test, at the $10 \%$ level of significance, whether or not the variances of the populations from which the samples are drawn are the same. State your hypotheses clearly.
(b) Calculate a 95\% confidence interval for the difference between the two mean milk yields.
(c) Explain the importance of the test in part (a) to the calculation in part (b).
4. A large number of students are split into two groups $A$ and $B$. The students sit the same test but under different conditions. Group A has music playing in the room during the test, and group B has no music playing during the test. Small samples are then taken from each group and their marks recorded. The marks are normally distributed.

The marks are as follows:

| Sample from Group A | 42 | 40 | 35 | 37 | 34 | 43 | 42 | 44 | 49 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sample from Group B | 40 | 44 | 38 | 47 | 38 | 37 | 33 |  |  |

(a) Stating your hypotheses clearly, and using a $10 \%$ level of significance, test whether or not there is evidence of a difference between the variances of the marks of the two groups.
(b) State clearly an assumption you have made to enable you to carry out the test in part (a).
(c) Use a two tailed test, with a $5 \%$ level of significance, to determine if the playing of music during the test has made any difference in the mean marks of the two groups. State your hypotheses clearly.
(d) Write down what you can conclude about the effect of music on a student's performance during the test.
5. The lengths, $x \mathrm{~mm}$, of the forewings of a random sample of male and female adult butterflies are measured. The following statistics are obtained from the data.

|  | No. of butterflies | Sample mean $\bar{x}$ | $\sum x^{2}$ |
| :---: | :---: | :---: | :---: |
| Females | 7 | 50.6 | 17956.5 |
| Males | 10 | 53.2 | 28335.1 |

(a) Assuming the lengths of the forewings are normally distributed test, at the $10 \%$ level of significance, whether or not the variances of the two distributions are the same. State your hypotheses clearly.
(b) Stating your hypotheses clearly test, at the $5 \%$ level of significance, whether the mean length of the forewings of the female butterflies is less than the mean length of the forewings of the male butterflies.
6. The weights, in grams, of apples are assumed to follow a normal distribution. The weights of apples sold by a supermarket have variance $\sigma_{S}{ }^{2}$. A random sample of 4 apples from the supermarket had weights

$$
\text { 114, 110, 119, } 123 .
$$

(a) Find a 95\% confidence interval for $\sigma_{S}{ }^{2}$.

The weights of apples sold on a market stall have variance $\sigma_{M}{ }^{2}$. A second random sample of 7 apples was taken from the market stall. The sample variance $s_{M}{ }^{2}$ of the apples was 318.8.
(b) Stating your hypotheses clearly test, at the 1\% level of significance, whether or not there is evidence that $\sigma_{M}{ }^{2}>\sigma_{S}{ }^{2}$.
7. Two machines $A$ and $B$ produce the same type of component in a factory. The factory manager wishes to know whether the lengths, $x$ cm, of the components produced by the two machines have the same mean. The manager took a random sample of components from each machine and the results are summarised in the table below.

|  | Sample Size | Mean $\bar{x}$ | Standard <br> Deviation $s$ |
| :---: | :---: | :---: | :---: |
| Machine $A$ | 9 | 4.83 | 0.721 |
| Machine $B$ | 10 | 4.85 | 0.572 |

The lengths of components produced by the machines can be assumed to follow normal distributions.
(a) Use a two tail test to show, at the 10\% significance level, that the variances of the lengths of components produced by each machine can be assumed to be equal.
(b) Showing your working clearly, find a $95 \%$ confidence interval for $\mu_{B}-\mu_{A}$, where $\mu_{A}$ and $\mu_{B}$ are the mean lengths of the populations of components produced by machine $A$ and machine $B$ respectively.

There are serious consequences for the production at the factory if the difference in mean lengths of the components produced by the two machines is more than 0.7 cm .
(c) State, giving your reason, whether or not the factory manager should be concerned.
(Total 13 marks)
8. A psychologist gives a test to students from two different schools, $A$ and $B$.

A group of 9 students is randomly selected from school $A$ and given instructions on how to do the test.

A group of 7 students is randomly selected from school $B$ and given the test without the instructions.

The table shows the time taken, to the nearest second, to complete the test by the two groups.

| $A$ | 11 | 12 | 12 | 13 | 14 | 15 | 16 | 17 | 17 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $B$ | 8 | 10 | 11 | 13 | 13 | 14 | 14 |  |  |

Stating your hypotheses clearly,
(a) test at the $10 \%$ significance level, whether or not the variance of the times taken to complete the test by students from school $A$ is the same as the variance of the times taken to complete the test by students from school B. (You may assume that times taken for each school are normally distributed.)
(b) test at the $5 \%$ significance level, whether or not the mean time taken to complete the test by students from school $A$ is greater than the mean time taken to complete the test by students from school $B$.
(c) Why does the result to part (a) enable you to carry out the test in part (b)?
(d) Give one factor that has not been taken into account in your analysis.
9. The random variable $X$ has a $\chi^{2}$-distribution with 9 degrees of freedom.
(a) Find $\mathrm{P}(2.088<X<19.023)$.

The random variable $Y$ follows an $F$-distribution with 12 and 5 degrees of freedom.
(b) Find the lower and upper 5\% critical values for $Y$.
10. The standard deviation of the length of a random sample of 8 fence posts produced by a timber yard was 8 mm . A second timber yard produced a random sample of 13 fence posts with a standard deviation of 14 mm .
(a) Test, at the $10 \%$ significance level, whether or not there is evidence that the lengths of fence posts produced by these timber yards differ in variability. State your hypotheses clearly.
(b) State an assumption you have made in order to carry out the test in part (a).
11. A mechanic is required to change car tyres. An inspector timed a random sample of 20 tyre changes and calculated the unbiased estimate of the population variance to be 6.25 minutes $^{2}$. Test, at the $5 \%$ significance level, whether or not the standard deviation of the population of times taken by the mechanic is greater than 2 minutes. State your hypotheses clearly.
(Total 6 marks)
12. A grocer receives deliveries of cauliflowers from two different growers, $A$ and $B$. The grocer takes random samples of cauliflowers from those supplied by each grower. He measures the weight $x$, in grams, of each cauliflower. The results are summarised in the table below.

|  | Sample size | $\sum x$ | $\sum x^{2}$ |
| :---: | :---: | :---: | :---: |
| $A$ | 11 | 6600 | 3960540 |
| $B$ | 13 | 9815 | 7410579 |

(a) Show, at the $10 \%$ significance level, that the variances of the populations from which the samples are drawn can be assumed to be equal by testing the hypothesis $\mathrm{H}_{0}: \sigma_{A}^{2}=\sigma_{B}^{2}$ against hypothesis $\mathrm{H}_{1}: \sigma_{A}^{2} \neq \sigma_{B}^{2}$.
(You may assume that the two samples come from normal populations.)

The grocer believes that the mean weight of cauliflowers provided by $B$ is at least 150 g more than the mean weight of cauliflowers provided by $A$.
(b) Use a 5\% significance level to test the grocer's belief.
(c) Justify your choice of test.
13. A beach is divided into two areas $A$ and $B$. A random sample of pebbles is taken from each of the two areas and the length of each pebble is measured. A sample of size 26 is taken from area $A$ and the unbiased estimate for the population variance is $S_{A}^{2}=0.495 \mathrm{~mm}^{2}$. A sample of size 25 is taken from area $B$ and the unbiased estimate for the population variance is $S_{B}^{2}=1.04$ $\mathrm{mm}^{2}$.
(a) Stating your hypotheses clearly test, at the $10 \%$ significance level, whether or not there is a difference in variability of pebble length between area $A$ and area $B$.
(b) State the assumption you have made about the populations of pebble lengths in order to carry out the test.

1. (a) $\mathrm{H}_{0}: \sigma_{1}^{2}=\sigma_{2}^{2}, \mathrm{H}_{1}: \sigma_{1}^{2} \neq \sigma_{2}^{2}$

B1
critical values $\mathrm{F}_{6,7}=3.87\left(\frac{1}{\mathrm{~F}_{6,7}}=0.258\right)$
B1
$\frac{s_{2}^{2}}{s_{1}^{2}}=\frac{5.2^{2}}{4.1^{2}} ;=1.61\left(\frac{s_{1}^{2}}{s_{2}^{2}}=\frac{4.1^{2}}{5.2^{2}}=0.622\right)$
M1; A1

Since 1.61 ( 0.622 ) is not in the critical region we accept $\mathrm{H}_{0}$ and conclude there is no evidence that the two variances are different
(b) $\mathrm{Sp}^{2}=\frac{7 \times 4.1^{2}+6 \times 5.2^{2}}{7+6}=21.53 \ldots$ $t_{13}=3.012$

$$
\begin{aligned}
99 \% \mathrm{CI}= & (17.9-15.9) \pm 3.012 \times \sqrt{21.53} \times \sqrt{\frac{1}{8}+\frac{1}{7}} \\
= & \pm(9.23,-5.233),[\text { or accept: }[0,9.23] \text { or }[-9.23,0]] \\
& \text { awrt } 9.23,-5.23
\end{aligned}
$$

(c) a person will be quicker at the task second time through/ times not independent/ familiar with the task/groups are not independent B1 1

## Note

B1 Allow $\sigma_{1}=\sigma_{2}$ and $\sigma_{1} \neq \sigma_{2}$
B1 must match their F

M1 for $\frac{s_{2}^{2}}{s_{1}^{2}}$

A1 awrt 1.61(0.622)
M1 A1 Sp ${ }^{2}$ may be seen in part a
B1 3.012 only
M 1 for $(17.9-15.9) \pm \mathrm{t}$ value $\times \sqrt{\mathrm{S}_{\mathrm{p}}^{2}} \times \sqrt{\frac{1}{8}+\frac{1}{7}}$
A1ft their $\mathrm{Sp}^{2}$
A1 awrt 9.23/-9.23 A1 awrt-5.23/5.23
(c) B1 any correct sensible comment
2. (a) $\mathrm{H}_{0}: \mu=70$ [accept $\left.\leq 70\right], \mathrm{H}_{1}: \mu>70$ B1
$t=\frac{71.2-70}{3.4 / \sqrt{20}}=1.58$ M1 A1
critical value $t_{19}(5 \%)=1.729$ B1
not significant, insufficient evidence to confirm manufacturer's claim

A1 ft

## Note

B1 both hypotheses using $\mu$
M1 $\frac{71.2-70}{3.4 / \sqrt{20}}$
A1 awrt 1.73
A1 correct conclusion ft their $t$ value and CV
(b) $\mathrm{H}_{0}: \sigma^{2}=16, \mathrm{H}_{1}: \sigma^{2} \neq 16$

B1

M1 A1

B1 B1

A1ft
6 gallon of the panther is different from that of the Tiger.

## Note

B1 both hypotheses and 16 . accept $\sigma=4$ and $\sigma \neq 4$
M1 $\frac{(19) \times 3.4^{2}}{16}$ allow $\frac{(19) \times 3.4^{2}}{4}$
A1 awrt 13.7
B1 32.852
B1 8.907
A1 correct contextual comment
NB those who use $\sigma^{2}=4$ throughout can get B0 M1 A0B1 B1 A1
3. (a) $\mathrm{H}_{0}: \sigma_{A}^{2}=\sigma_{B}^{2}, \mathrm{H}_{1}: \sigma_{A}^{2} \neq \sigma_{B}^{2}$
critical values $\mathrm{F}_{12,8}=3.28$ and $\frac{1}{F_{8,12}}=0.35$
$\frac{s_{B}^{2}}{s_{A}^{2}}=2.40\left(\frac{s_{A}^{2}}{s_{B}^{2}}=0.416\right)$
Since 2.40 ( 0.416 ) is not in the critical region we accept $\mathrm{H}_{0}$ and conclude there is no evidence that the two variances are different.
(b) $\mathrm{Sp}^{2}=\frac{8 \times 1.02+12 \times 2.45}{20}$
$=1.878$
$(27.94-25.54) \pm 2.086 \times \sqrt{1.878} \times \sqrt{\frac{1}{9}+\frac{1}{13}}$
(1.16, 3.64)

B1
(c) To calculate the confidence interval the variances need to be equal. B1

In part (a) the test showed they are equal.
B1 2
4.
(a) $\quad \mathrm{H}_{1}: \sigma_{\mathrm{A}}^{2}=\sigma_{\mathrm{B}}^{2} \quad \mathrm{H}_{0}: \sigma_{\mathrm{A}}^{2} \neq \sigma_{\mathrm{B}}^{2}$
B1
$\mathrm{S}_{\mathrm{A}}{ }^{2}=22.5 \quad \mathrm{~S}_{\mathrm{B}}{ }^{2}=21.6$
awrt M1A1A1
$\frac{s_{1}^{2}}{s_{2}^{2}}=1.04$
M1A1
$F_{(8,6)}=4.15$
B1
$1.04<4.15$ do not reject $\mathrm{H}_{0}$. The variances are the same.
B1 8
(b) Assume the samples are selected at random, (independent)
B1 1
(c) $s_{p}^{2}=\frac{8(22.5)+6(21.62)}{14}=22.12$
awrt 22.1 M1A1
$\mathrm{H}_{0}: \mu_{\mathrm{A}}=\mu_{\mathrm{B}} \quad \mathrm{H}_{1}: \mu_{\mathrm{A}} \neq \mu_{\mathrm{B}}$ B1
$t=\frac{40.667-39.57}{\sqrt{22.12} \sqrt{\frac{1}{9}+\frac{1}{7}}}$
$=0.462 \quad 0.42-0.47$
A1
Critical value $=t_{14}(2.5 \%)=2.145$
B1
$0.462<2.145$ No evidence to reject $H_{0}$. The means are the same
B1
(d) Music has no effect on performance
B1 1
[17]
5. (a) $\mathrm{H}_{0}: \sigma_{\mathrm{F}}^{2}=\sigma_{\mathrm{M}}^{2} \quad \mathrm{H}_{1}: \sigma_{\mathrm{F}}^{2} \neq \sigma_{\mathrm{M}}^{2}$

B1
$s^{2}{ }_{F}=\frac{1}{6}\left(17956.5-7 \times 50.6^{2}\right)=\frac{33.98}{6}=5.66333 \ldots$
B1
$s^{2}{ }_{M}=\frac{1}{9}\left(28335.1-10 \times 53.2^{2}\right)=\frac{32.7}{9}=3.63333 \ldots$
B1
$\frac{s^{2}{ }_{F}}{s^{2}{ }_{\mathrm{M}}}=1.5587 \ldots$ (Reciprocal 0.6415)
M1A1
$\mathrm{F}_{6,9}=3.37$ ( or 0.24)
Not in critical region. Variances of the two distributions are the same A1 7 need to have variance and the same o.e.

```
(b) \(\mathrm{H}_{0}: \mu_{\mathrm{F}}=\mu_{\mathrm{M}} \quad \mathrm{H}_{1}: \mu_{\mathrm{F}}<\mu_{\mathrm{M}} \quad\) B1
Pooled estimate \(s^{2}=\frac{6 \times 5.66333 \ldots+9 \times 3.63333}{15}\)
\(=4.44533\)
\[
s=2.11
\]
\[
\mathrm{t}=\frac{50.6-53.2}{2.11 \sqrt{\frac{1}{7}+\frac{1}{10}}}= \pm 2.50
\]
C.V. \(\mathrm{t}_{15}(5 \%)= \pm 1.753\)
```

B1
A1 6

```
Significant. The mean length of the females forewings is less than the length of the males forewings need female and forewing(wing)
```

6. (a) $\left(\bar{x}=\frac{466}{4}=116.5\right)$
$S_{x^{2}}=\frac{54386-4 \bar{x}^{2}}{3}=32.3$ or $\frac{97}{3}$ or awrt 32.3
M1 A1
$0.216<\frac{3 S_{x^{2}}}{\sigma^{2}}<9.348$
B1 M1 B1
10.376... $<\sigma^{2}<449.07 \ldots$
awrt 10.4, 449
A1 A1 7
(b)
$\mathrm{H}_{0}: \sigma_{m}=\sigma_{\mathrm{s}} \quad \mathrm{H}_{1}: \sigma_{m}>\sigma_{\mathrm{s}}$
$\mathrm{H}_{0}: \sigma^{2}=\sigma^{2} \quad \mathrm{H}_{1}: \sigma^{2}>\sigma_{s}^{2}$
$\frac{S_{m^{2}}}{S_{s^{2}}}=\frac{318.8}{32.3}=9.859 .$.
awrt 9.86
M1 A1
$\mathrm{F}_{6.3}\left(17_{0}\right.$ c.v. $)=27.91$ B1
$9.15<27.91 \quad$ A1ft
Insufficient evidence of an increase in variance
Insufficient evidence to say $\sigma_{m}{ }^{2}>\sigma_{s}{ }^{2}$ is OK Variance can be assumed to be the same is OK
[NB $\frac{32.3}{318.3}=0.101 \ldots$ only gets M1A1 if appropriate $F$ value attempted.]
7. (a) $\left(\mathrm{H}_{0}: \sigma_{\mathrm{A}}^{2}=\sigma_{\mathrm{S}}^{2} \quad \mathrm{H}_{1}: \sigma_{\mathrm{A}}^{2} \neq \sigma_{\mathrm{S}}^{2}\right)$
$\frac{\mathrm{S}_{A}^{2}}{\mathrm{~S}_{S}^{2}}=\frac{0.721^{2}}{0.572^{2}}=1.588 \ldots \quad$ awrt $1.59 \quad$ M1 A1
$\mathrm{F}_{8.9}(5 \%)$ c.v. $[=10 \% 2$ tail $]=3.23$ B1

B1 c.a.o. 4

$$
\left(\text { accept } \sigma_{A}^{2}=\sigma_{S}^{2}\right)
$$

(b) $\mathrm{Sp}^{2}=\frac{8 \times 0.721^{2}+9 \times 0.572^{2}}{8+9}=0.41784 \ldots \quad$ M1 A1

$$
0.417 \ldots \text { or awrt } 0.418
$$

$\mathrm{t}_{17}=(2.5 \%)$ c.v. $=2.110 \quad$ B1
95\% C.I. $\quad=\overline{\mathrm{x}}_{\mathrm{B}}-\overline{\mathrm{x}}_{\mathrm{A}} \pm 2.110 \times \mathrm{Sp} \times \sqrt{\frac{1}{9}+\frac{1}{10}} \quad$ M1
$=0.02 \pm 2.110 \times \sqrt{0.417 \ldots} \times \sqrt{\frac{1}{9}+\frac{1}{10}}$
$=(-0.6066 \ldots, 0.6466 \ldots) \quad$ awrt $(-0.607,0.647) \quad$ A1, A1 7
(c) $\pm 0.7$ is outside interval
$\therefore$ Manager need not be concerned (allow ft if 0.7 inside)
(dep) B1ft
2
8. (a) $s_{\mathrm{A}}{ }^{2}=5.11,5_{\mathrm{B}}{ }^{2}=5.14$ B1 B1
$\mathrm{H}_{0}: \sigma_{\mathrm{A}}^{2}, \mathrm{H}_{1}: \sigma_{\mathrm{A}}^{2} \neq \sigma_{\mathrm{B}}^{2}$ B1

Critical value $\mathrm{F}_{6}, 8=3.58$

$$
\frac{s_{\mathrm{B}}{ }^{2}}{s_{\mathrm{A}}{ }^{2}}=1.0062112 \ldots
$$

awrt 1.01
M1 A1

No evidence to reject $\mathrm{H}_{0}$. The variances are equal.
(b) $\quad s_{\mathrm{p}}{ }^{2}=\frac{8 \times 5.14+6 \times 5.11}{9+7-2}=5.1247$
awrt $5.12 \quad$ M1 A1
$\mu_{\mathrm{A}}=14.11 . ., \mu_{\mathrm{B}}=11.857$..
$\mathrm{H}_{0}: \mu_{\mathrm{A}}=\mu_{\mathrm{B}}, \mathrm{H}_{1}: \mu_{\mathrm{A}}>\mu_{\mathrm{B}}$
B1
Critical value $t_{14}(5 \%)=1.761$
B1
$T=\frac{14.11 . .-11.857 \ldots}{\sqrt{5.1247 \ldots\left(\frac{1}{9}+\frac{1}{7}\right)}}=1.9757$
awrt 1.98 M1 A1

There is evidence to reject $\mathrm{H}_{0}$.
Mean time taken from school A is greater than school B.
A1 7
(c) Equal variances are a condition for the test in part(b)

B1 1
(d) Groups not equal ability

B1 1
9. (a) $\mathrm{P}(X>19.023)=0.025$ or $\mathrm{P}(X<19.023)=0.975$ both B1
$\mathrm{P}(X>2.088)=(0.990$ or $\mathrm{P}(X<2.088)=0.010$
$\therefore \mathrm{P}(2.088<X<19.023)=0.990-0.025$ or $0.975-0.010$ $=\underline{0.965}$
(b) Upper Critical value of $\mathrm{F}_{12,5}=4.68$ B1

Lower Critical value of $\mathrm{F}_{12,5}=\frac{1}{\mathrm{~F}_{5,12}}$ M1

$$
=\frac{1}{3.11}=0.3215 \ldots
$$

A1 3
awrt 0.322
10. (a) $\mathrm{H}_{0}: \sigma_{2}^{2}=\sigma_{2}^{2} ; \mathrm{H}_{1}: \sigma_{1}^{2} \neq \sigma_{2}^{2}$
C.V. $\mathrm{F}_{12,7}=3.57$
C.V.: $F_{7,12}=\frac{1}{3.57}=0.28011$

B1

Since 3.0625 is not in the Critical Region there is insufficient evidence to reject $\mathrm{H}_{0}$. There is insufficient evidence of a difference in the variances of the length of the fence posts.
(b) The distribution of the population of lengths of fence posts is normally distributed.

B1 1
11. $\mathrm{H}_{0}: \sigma^{2}=4 ; \mathrm{H}_{1}: \sigma^{2}>4$
both

$$
v=19, \quad X_{19}^{2}(0.05)=30.144
$$

$$
\frac{(n-1) S^{2}}{\sigma^{2}}=\frac{19 \times 6.25}{4}=29.6875
$$

$$
\text { use of } \frac{(n-1) S^{2}}{\sigma^{2}}
$$

M1
AWRT 29.7

## A1

Since $29.6875<30.144$ there is insufficient evidence to reject $\mathrm{H}_{0}$.
A1 ft
There is insufficient evidence to suggest that the standard deviation is greater than 2 .

B1 ft
12. (a) $S_{A}^{2}=\frac{1}{10}\left\{3960540-\frac{6600^{2}}{11}\right\}=\underline{54.0}$
$S_{B}^{2}=\frac{1}{12}\left\{7410579-\frac{9815^{2}}{13}\right\}=\underline{21.1 \dot{6}}$
B1
$\mathrm{H}_{0}: \sigma_{A}^{2}=\sigma_{B}^{2} ; \mathrm{H}_{1}: \sigma_{A}^{2} \neq \sigma_{B}^{2}$ B1
CR: $\mathrm{F}_{10,12}>2.75$
$S_{A}^{2} / S_{B}^{2}=\frac{54.0}{21.1 \dot{6}}=2.55118 \ldots$
Since $2.55118 \ldots$ is not in the critical region, we can assume that the variances of $A$ and $B$ are equal.
(b) $\mathrm{H}_{0}: \mu_{B}=\mu_{A}+150 ; \mathrm{H}_{1}: \mu_{B}>\mu_{A}+150$

B1
both
CR: $t_{22}(0.05)>1.717$
B1
$S_{p}^{2}=\frac{10 \times 54.0+12 \times 21.1 \dot{6}}{22}=\underline{36.09 \dot{0} \dot{9}}$
M1 A1
$t=\frac{1755-6001-150}{\sqrt{36.0909 \ldots\left(\frac{1}{11}+\frac{1}{13}\right)}}=\underline{2.03157}$
M1 A1

AWRT 2.03
A1
Since $2.03 \ldots$ is in the critical region we reject $\mathrm{H}_{0}$ and conclude that the mean weight of cauliflowers from $B$ exceeds that from $A$ by at least 150 g .

A1 ft 8
(c) Samples from normal populations

B1 B1
2
Any two sensible verifications
Equal variances
Independent samples
[16]
13. (a) $\mathrm{H}_{0}: \sigma_{A}^{2}=\sigma_{B}^{2}, \mathrm{H}_{1}: \sigma_{A}^{2} \neq \sigma_{B}^{2}$
both
B1
critical values $\mathrm{F}_{24.25}=1.96$ and $\frac{1}{\mathrm{~F}_{24.25}}=0.510$ both $\frac{s_{B}^{2}}{s_{A}^{2}}=2.10$ or $\frac{s_{A}^{2}}{s_{B}^{2}}=0.476 \quad$ both B1

Since 2.10 or 0.476 are in the critical region we reject $\mathrm{H}_{0}$ and conclude there is evidence of a difference in variability of pebble length between area $A$ and $B$

A1] 5
(b) The populations of pebble lengths are normal.

B1 1

1. This proved to be a good starter question and most candidates gave good solutions. Part (a) was answered well with many candidates gaining full marks.

In part(b) although a pooled estimate was worked out correctly by many candidates they then failed to use the square root of it in their calculations of the confidence interval or they used $\sqrt{\frac{21.53}{15}}$ rather than $\sqrt{21.53\left(\frac{1}{8}+\frac{1}{7}\right)}$. A few candidates found the confidence intervals for the mean times separately rather than for the difference.
2. The most able candidates gained full marks for this question. In part (b) a minority of candidates tested whether the variances of the two cars were equal rather than testing whether the variance of the Panther car was equal to 16.
3. Part (a) was answered well with many candidates gaining full marks. In part (b) although a pooled estimate was worked out correctly by many candidates they then failed to use the square root of it in their calculations of the confidence interval.

In part (c) candidates knew that to find the confidence interval/pooled estimate that the variances needed to be equal but few commented on the fact that this has been established in part (a).
4. This question was answered well with a large proportion of the candidates getting full marks. In part (b) many candidates stated that the marks were normally distributed which was not an assumption they had made as it had been stated in the question.
In part (c) the pooled estimate was worked out correctly by many candidates but they then failed to use the square root of it in their calculations of $t$. Many candidates were able to draw a conclusion in the context of the question.
5. This question was well answered with a large proportion of the candidates getting full marks. In part (b) the pooled estimate was worked out correctly by many candidates but they then failed to use the square root of it in their calculations of $t$. Many concluded that females were shorter than males rather than it being the forewing length which was shorter
6. Most realized that the Chi squared distribution was required to establish the confidence interval in part (a) and there were many correct solutions. The $F$ test was usually used in part (b) but sometimes the degrees of freedom were the wrong way around and some used a $5 \%$ significance level.
7. $\quad$ The two tailed $F$ test was usually tackled quite well but the confidence interval in part (b) was not. A pooled estimate of variance $s_{p}{ }^{2}$ was required and this was often attempted but the $\sqrt{\frac{1}{9}+\frac{1}{10}}$ term sometimes divided into $s_{p}$ rather than multiplying it. The interpretation in part(c) was often answered well and the follow through enabled those who could interpret the statistics to gain some credit.
8. Candidates found this question very demanding and parts (a) and (b) were usually confused and incoherent due to accuracy errors.
9. Many candidates were able to answer this question correctly but too many showed that they had not understood the $F$-distribution tables. A clear shaded and labelled diagram would have helped many candidates.
10. Although generally well answered common errors were the division of the standard deviations; confusing the degrees of freedom and hence finding the wrong critical value; not giving the conclusion in context and not making reference to the population in the final part.
11. This question was generally very well answered. Weaker candidates sometimes confused the unbiased estimate of the population variance with the given population value and they often forgot to relate their conclusion to the context of the question.
12. Many of the candidates scored full marks in part (a). In part (b) too many of them could not specify the hypotheses correctly since they did not know how to cope with the 150 g . Surprisingly many of the candidates did not score both marks in the final part of this question.
13. The question proved to be a friendly starter for the majority, with many candidates gaining full marks. The most common error was in the conclusion where the context was omitted.

